A Case Study on the Effectiveness of LLMs in Verification with Proof Assistants

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Abstract

Large language models (LLMs) can potentially help with verification using proof assistants by automating proofs. However, it is unclear how effective LLMs are in this task. In this paper, we perform a case study based on two mature Rocq projects: the `hs-to-coq` tool and Verdi. We evaluate the effectiveness of LLMs in generating proofs by both quantitative and qualitative analysis. Our study finds that: (1) external dependencies and context in the same source file can significantly help proof generation; (2) LLMs perform great on small proofs but can also generate large proofs; (3) LLMs perform differently on different verification projects; and (4) LLMs can generate concise and smart proofs, apply classical techniques to new definitions, but can also make odd mistakes.

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1 Introduction

Software should be correct. But in reality, that's rarely true.

Proof assistants allow us to formally *verify* that a class of bugs is *absent* in a program via mechanized mathematical proofs. In the past two decades, various works have demonstrated that this approach is a feasible way to ensure software correctness and reliability. Some notable verified software includes the CompCert C compiler [45], the seL4 microkernel [42], the CertiKOS operating system [30], the FSCQ file system [12], *etc.* Many new tools and frameworks that support mechanized reasoning have also emerged, including program logics and frameworks for reasoning about concurrency [10, 40, 59, 79], nonterminating programs [75, 89], nondeterminism [11, 17, 60], *etc.*

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However, despite all these efforts, proving the correctness theorems of a program in a proof assistant remains a daunting task. For example, Breitner et al. [9]'s work on verifying Haskell's `containers` library using `hs-to-coq` shows that their verification work "required 8.9 lines of proof per lines of code" [9, Section 3]. This is a significant overhead in addition to code development. Future changes to the code or the specification bring even greater challenges for proof maintenance and proof repair [28, 70, 71].

Large Language Models, or LLMs, on the other hand, have received great attention for their capability in performing a wide range of tasks. In particular, existing works have demonstrated LLMs' effectiveness in generating code [38] and mathematical proofs [2, 46, 83].

It is natural to ask: Can LLMs help with verification using proof assistants?

Indeed, researchers have recently started investigating this question, and various new tools/frameworks for generating program correctness proofs with the help of LLMs have also emerged [25, 44, 51, 67]. However, due to the mysterious nature of LLMs [100, 102], many questions remain unanswered.

In this paper, we build on prior works and try to understand more about the effectiveness of LLMs in verification with proof assistants, by conducting a case study on two different verification projects that use Rocq Prover [81]: the `hs-to-coq` tool [9, 78] and Verdi [86, 87].

Our case study investigates the following research questions:

- **RQ1:** How do external dependencies and/or context in the same source file impact proof generation for a theorem?
- **RQ2:** How do LLMs perform on proofs of different sizes?
- **RQ3:** Is there a difference when running LLMs on different verification projects?
- **RQ4:** How is the quality of proofs generated by LLMs?

To answer these questions, we conduct a *quantitative* study for RQ1, RQ2, and RQ3, and a *qualitative* study for RQ4. Our case study shows:

- Including either external dependencies or context in the same source file, or both, can significantly improve the effectiveness of LLMs in generating proofs.

- LLMs perform significantly better on proofs of smaller sizes. However, there is still a chance for LLMs to generate proofs consisting of more than 20 tactics.
- LLMs perform differently in the two projects we studied. For example, LLMs are less likely to generate proofs that are identical to original proofs in `hs-to-coq`. Context in the same source file also plays a more significant role in generating proofs for Verdi.
- LLMs can generate concise and smart proofs. They can also apply classical techniques such as performing a case analysis on an inductively defined proposition. On the other hand, LLMs can also generate odd, apparently failed proofs that repeat a tactic seemingly indefinitely.

In the rest of this paper, we first introduce some necessary background about Rocq Prover and program verification in Section 2. We then discuss our target codebase, namely, `hs-to-coq` and Verdi, in Section 3. We describe our methodology in Section 4. We share and interpret our results (both quantitative and qualitative ones) in Section 5. We discuss related works in Section 6. Finally, we conclude with Section 7.

2 Background

In this section, we introduce the necessary background knowledge to understand this paper. Readers who are familiar with these concepts should feel free to skip the relevant parts.

Rocq Prover. Some commonly used proof assistants include Rocq Prover [81], Agda [1], Lean [16], F⋆ [80], and Isabelle [58], *etc.* In this paper, we focus on Rocq Prover. Rocq Prover is formerly known as the Coq proof assistant[1] and is one of the most commonly used proof assistants for program verification. Rocq Prover has an expressive specification language and supports full dependent types, which enables describing the properties of a software system in rich detail.

We illustrate the process of theorem proving in Rocq Prover in Fig. 1. We first state a theorem in Rocq Prover using its specification language. For example, line 1 of Fig. 1a is equivalent to the mathematical proposition:

$$\forall n \in \mathbb{N}, n + 0 = n$$

where $\mathbb{N}$ is the set of all natural numbers.

We can then write a *proof script* that instructs the proof assistant to prove this theorem, as shown in lines 2–7. However, in Rocq Prover, we typically do not directly write the entire proof script. Instead, we enter an *interactive* proof mode. This step is typically marked by the `Proof` keyword (line 2).

When we enter the proof mode, Rocq prover will display the current context and the proof goal as shown in Fig. 1b. The *context*, which is empty at this point, consists of all the current hypotheses. The *proof goal* is what we need to show to finish the proof. We can manipulate the context and the proof goal using *tactics*, which are instructions to Rocq Prover about how to proceed with the proof. In this case, we decide to do an induction over n, indicated by the tactic `induction n` (line 3 in Fig. 1a). Our tactic also names some new variables via an *intro pattern* `[ | n' IHn' ]`—these names will show up later in the proof process.

After invoking the `induction` tactic, our proof goal will become two subgoals: one for the base case and one for the induction step. Rocq Prover will first ask us to prove the base case. We show the context and the goal of the base case in Fig. 1c. We can see that the context is still empty, but the goal has been changed to prove that 0 + 0 = 0. Rocq Prover can

```
1 Theorem add_0_r : forall n:nat, n + 0 = n.
2 Proof.
3   induction n as [ | n' IHn'].
4   - (* n = 0 *)    reflexivity.
5   - (* n = S n' *) simpl. rewrite -> IHn'.
6     reflexivity.
7 Qed.
```

(a) A Rocq theorem about natural numbers and its proof. The example comes from *Logical Foundations*, a classical textbook on Rocq Prover [66].

```
------------------------------
forall n : nat, n + 0 = n
```

(b) The context and the proof goal when we enter the proof mode (*i.e.,* right after invoking line 2).

```
------------------------------
0 + 0 = 0
```

(c) The context and the proof goal after we invoke the `induction` tactic (*i.e.,* after line 3). This is the first goal, *i.e.,* the base case.

```
n' : nat
IHn' : n' + 0 = n'
------------------------------
S n' + 0 = S n'
```

(d) The context and the proof goal after we proved the base case (*i.e.,* after line 4). This is the induction step.

```
n' : nat
IHn' : n' + 0 = n'
------------------------------
S n' = S n'
```

(e) The context and the proof goal after the `rewrite` tactic (*i.e.,* after line 5).

Figure 1. The process of proving a theorem in Rocq Prover.

[1]The name change starts in Rocq Prover version 9.0. However, we will address all versions of Rocq Prover, including those before this name change, as Rocq Prover to avoid confusion.

tell that $0+0$ computes to 0, so proving the goal is equivalent to proving that $0 = 0$. Rocq Prover knows that = is *reflexive*, so we can discharge this goal via the `reflexivity` tactic.

Once we are done with the base case, Rocq Prover will ask us to prove the induction step. We show the context and the goal of the induction step in Fig. 1d. This time, the context contains a variable `n'` that has type `nat` and an *induction hypothesis* `IHn'` that states `n' + 0 = n'`. The names `n'` and `IHn'` come from the intro pattern in our `induction` tactic earlier. Our goal has also been changed to show that `S n' + 0 = S n'`, where `S n'` means the successor of `n'`.

In Rocq, `S n' + 0` is recursively defined as `S (n' + 0)`. We can reveal this via the `simpl` tactic. After that, we recognize that `n' + 0` equals `n'` by the induction hypothesis `IHn'`, so we can `rewrite` using `IHn'`. The `rewrite` changes the goal to Fig. 1e, which can again be solved by the `reflexivity` tactic.

Finally, we can write a **`Qed`** at the end of the proof (line 7 in Fig. 1a). **`Qed`** is more than an end mark of a proof in Rocq Prover: it checks that the proof constructed by our proof script is indeed a correct proof of the theorem. Rocq Prover has a small trusted computing base for proof checking. This means that a proof checked by **`Qed`** is highly trustworthy.

Program Verification with Rocq Prover. We can verify properties of a program in the same way in Rocq Prover. If the program has already existed but was written in another language, we need to first *embed* the syntax and/or semantics of that program in Rocq Prover [8]. Alternatively, we can also write a program directly in Rocq, prove properties about it, and then *extract* it to another language like OCaml or C. Our evaluation includes examples in both approaches (Section 3).

There have been various verification works based on Rocq Prover, including compilers [45, 96], operating systems [30], file systems [12], networked servers [43, 97], cryptographic algorithm implementations [22], *etc.* There are also various tools/frameworks supporting program verification with Rocq Prover, including Verified Software Toolchains [3], Iris [40, 79], certified abstraction layers [29, 31], mathematical components [52], and interaction trees [89, 95], *etc.* A more detailed account of Rocq Prover's ecosystem can be found in Appel [4].

3 Codebase for Evaluation

We chose two open-source Rocq Prover projects as the evaluation target: (1) the theory for Haskell's `base` library contained in the `hs-to-coq` project [9, 77, 78], and (2) the Verdi project for implementing and verifying distributed systems [86, 87].

We chose these two projects for the following reasons:

First, these two projects represent the two most typical ways to represent programs: functions and inductively defined relations. Programs in the `hs-to-coq` codebase are all purely functional programs, so they are simply defined as Rocq functions.[2] The Verdi project, on the other hand, reasons about traces and transition systems encoded by inductively defined propositions.

```
take                    :: Int -> [a] -> [a]
take n _      | n <= 0 =  []
take _ []               =  []
take n (x:xs)           =  x : take (n-1) xs
```

(a) The **`take`** function defined in Haskell's `base` library.

```
Fixpoint take {a:Type} (n:Z) (xs:list a) :
  list a :=
  if (n <=? 0)%Z then nil
  else match xs with
       | nil => nil
       | cons y ys => cons y (take (n - 1) ys)
       end.
```

(b) The **`take`** function converted to Rocq by `hs-to-coq`. The **`Fixpoint`** keyword marks the definition of a recursive function. Haskell's types, such as **`Int`** and lists `[]`, are translated to Rocq types `Z` and `list`.

Figure 2. The **`take`** function defined in Haskell's `base` library and its translation in Rocq Prover produced by `hs-to-coq`.

Second, these two projects have proper sizes for an initial investigation of LLMs' effectiveness in verification. On the one hand, they are no toy projects. The theory of `base` in `hs-to-coq` contains 187 proofs, and Verdi contains 579 proofs. On the other hand, these projects are not too large.

Finally, none of these projects involve advanced program logics (*e.g.,* separation logic [69], concurrent separation logic [10, 59]) or frameworks (*e.g.,* certified abstraction layers [29, 31], interaction trees [89, 95]). The absence of these advanced reasoning tools helps keep the experiments pristine.

We now talk about each project and the part we evaluate in more detail.

3.1 The `hs-to-coq` Project

The `hs-to-coq` tool translates purely functional programs in Haskell to a shallow embedding in Rocq Prover. Its open-source repository contains translated code from Haskell's `base` library, `containers` library, parts of the GHC compiler, and many other examples of different sizes. We show an example of the original Haskell code and the translated Rocq code in Fig. 2. More details on how such a translation works can be found in Spector-Zabusky et al. [78].

Our case study is based on the translated Rocq code, and theorems stated and proven by the `hs-to-coq` developers. Once a piece of Haskell code is translated using `hs-to-coq`,

[2] They should be called Gallina functions, to be more precise. Gallina is the specification language of Rocq Prover. However, we will not try to intentionally distinguish Gallina and Rocq in this paper.

```
Class EqLaws (t : Type) `{Eq_ t} :=
  { Eq_refl  : reflexive  _==_;
    Eq_sym   : symmetric  _==_;
    Eq_trans : transitive _==_;
    Eq_inv   : forall x y : t, x == y = ~~ (x /= y)
  }.

Class EqExact (t : Type) `{EqLaws t} :=
  { Eq_eq : forall x y : t, reflect (x = y) (x == y) }.
```

Figure 3. Laws for the **Eq** typeclass stated in Rocq Prover in hs-to-coq.

we can treat the translated code as regular Rocq code, so our evaluation does not rely on the hs-to-coq tool or any Haskell code.

Our case study focuses on the theory of the base library. The base library contains a number of basic Haskell types, functions, typeclasses, and typeclass instances. The theory of base contains theorems for these basic types and functions, and theorems for typeclass laws.

Typeclasses are a way to implement *overloading* (*i.e., ad-hoc polymorphisms*) in functional languages, including both Haskell and Rocq Prover [34, 76, 85]. A few examples of typeclasses implemented in the base library include: **Eq** for equality tests, **Ord** for total orders, Semigroup for concatenation, Foldable for "congregating" a data structure, and abstract interfaces like **Functor**, Applicative [53], and **Monad** [55, 84], *etc.*

Instances of these typeclasses are expected to satisfy certain laws. For example, an implementation of equality tests == in **Eq** should be reflexive, transitive, and symmetric; the <= operator in **Ord** should be reflexive, transitive, and antisymetric; a **Monad** should satisfy monad laws [55, 84]. The documentation of the base library describes these laws in details.

We show an example of how hs-to-coq's theory of base states laws for the **Eq** typeclass in Fig. 3. These laws are themselves defined as typeclasses in Rocq Prover. Eq_refl, Eq_sym, and Eq_trans state that == is reflexive, symmetric, and transitive, respectively. In hs-to-coq, _==_ represents the equality test function, and == is a notation that can be used as an infix operator. Eq_inv states that == and /= are inverse of each other. Finally, EqExact contains a special law that states == always agrees with Rocq's builtin equality =. The reflect definition is an interesting definition that enables a classical technique in mechanized reasoning called *proof by reflection*. We will see an example of LLMs using this later in Section 5.

We choose the theory of base because it contains a fair amount of theorems, and the proofs in general are neither too simple nor too complicated. The longest proof script involves 43 tactics.

Other theories, such as theories for containers, graph, or the GHC compiler, contain much more complicated proofs. For example, the theorem insertBM_Desc is about the property of the insertBM function of container's IntSet data structure.[3] The handcrafted proof of this theorem is 42 lines of proof script, makes heavy use of proven lemmas, uses custom tactics, uses Ltac's match clause for pattern matching certain goals to solve them automatically, involves both backward reasoning and forward reasoning using assert. We leave the investigation of these examples to future work.

The hs-to-coq project relies on Rocq Prover 8.10, which is an old version first released in April 2019. Unfortunately, most of its code no longer works under later versions of Rocq Prover because Rocq Prover does not support backward compatibility. For this reason, we conduct our study on Rocq Prover 8.10 as well. This should not impact the validity of this research, as the key workflow and features of Rocq Prover remain the same across these versions.

3.2 Verdi

Verdi is a framework for implementing and verifying distributed systems in Rocq Prover. Instead of writing a program in a different language and embedding it in Rocq Prover, a programmer first implements their distributed systems in Rocq Prover and extract the code to OCaml. Unlike purely functional programs in Haskell's base library, distributed systems always contain a number of effects and interact with a network that can reorder or even drop messages. To model this, Verdi defines a special monad for implementing distributed systems and transition systems for network semantics. More details about how Verdi works can be found in Wilcox et al. [86], Woos et al. [87].

The Verdi framework has been used in various works to study the effectiveness of AI in verification. For example, it is included as part of the CoqGym benchmark [92] and has been studied by First and Brun [23], First et al. [24]. In particular, Lu et al. [51] tried applying GPT-3.5[4] to proofs in Verdi. They found that LLMs like GPT-3.5 are ineffective in finishing most of the proofs, as they collected 520 errors out of 579 theorems. They further analyzed all the errors and made the following observation [51, Section 3]:

> …while LLMs often generate proof scripts with the right high-level structure, they often struggle with accurately addressing the sorts of low-level details that hammers excel at. For example, GPT-3.5 often knows when to use the induction tactic to decompose theorems into subgoals, but often fails to generate the right sequence of tactics to prove each subgoal…

This paper builds on these prior studies, but also investigates the effectiveness of dependencies in prompting.

[3] The data structure is a Patricia trie [56, 61].
[4] https://platform.openai.com/docs/models/gpt-3.5-turbo

The Verdi project we experiment with is the version included in CoqGym [92] and relies on Rocq Prover 8.11, to be consistent with prior studies.

4 Methodology

To evaluate how models performed under different contexts, we extracted the following information for each top-level construct using SerAPI [26] version 8.10.0+0.7.2 for `hs-to-coq`, and 8.11.0+0.11.1 for Verdi[5], along with Rocq version 8.10.2 and 8.11.0, respectively:

Name and signature: For each top-level definition in a Rocq source file, we extracted its name (*i.e.,* the identifier bound by the construct) and its signature. For theorems, the signature consists of the entire declaration excluding the proof. For other definitions, the signature includes the entire definition.

In-file context: We defined the in-file context as all lines in the file prior to the location where a theorem appears.

External dependencies, or dependencies: We defined external dependencies (or dependencies for short) as any *signatures* that the original proof relies on, including definitions and theorems from other source files. If a dependency was already included in the in-file context, we excluded it from the list of dependencies to avoid repetition. Our extraction may include unnecessary dependencies. Specifically, qualified identifiers returned by SerAPI can match identifiers defined in multiple files. In such cases, we included all matching possibilities in the dependency list.

Notations: For each dependency and file imported via Rocq's `Require Import` command, we collected all associated notation declarations. However, the definitions underlying these notations were not necessarily included as dependencies, since a notation may be used without its underlying definition being required by the proof.

Model and parameter selection. Our model selection includes both general-purpose and reasoning models with a mix of full-sized and lightweight variants:

1. GPT-4o-mini, version 2024-07-18: A smaller general-purpose model with a context length of 128,000 tokens [63].
2. GPT-4o, version 2024-11-20: A general-purpose model with a context length of 128,000 tokens [62].
3. OpenAI o4-mini, version 2025-04-16: A smaller reasoning model with a context length of 200,000 tokens [64]. The model does not support changing the default temperature through the API, but supports a reasoning effort parameter [54]. For our experiments, we have selected reasoning effort 'medium,' which is the default.
4. DeepSeek Prover V2: An open-source model based on DeepSeek V3. This model is fine-tuned for theorem proving in Lean 4. The model has a context length of 163,840 tokens [90] and a parameter count of 671 billion [68]. We include this model in our case study to check if exposure to mechanized proofs in another proof assistant transfers to Rocq proofs.
5. DeepSeek R1: A large open-source reasoning model with a context length of 163,840 tokens [91], and a parameter count of 671 billion [18].

Each model was prompted with the same system message (for models supporting system prompt), and was allowed a maximum of 16,384 output tokens, configured using `max_tokens` or `max_completion_tokens` based on the model. The original context lengths for each model were preserved.

For all experiments, we set the temperature to 0.1 for models that support modifying this parameter over the API (e.g., GPT-4o). For models that do not support a custom temperature setting (e.g., o4-mini), the default value of 1.0 was used.

Prompt. We used a minimal system prompt that described (1) the information provided to the model, (2) the proof task it has to perform, and (3) the expected response format, asking the model to respond only with the proof body. The prompt also specified the current version of the Rocq available and included whether the version used `omega` in place of `lia`. We included this detail as the codebases being evaluated were relatively old, whereas the models, which have more recent knowledge cutoffs, are likely aware that `omega` is deprecated.

Variation of dependencies. We varied the prompt provided to the LLMs across four conditions: (1) full context (which we will shorten as *the informed mode* from now on), (2) without dependencies and notations, (3) without in-file context, and (4) with both removed.

When omitting the in-file context, we still include the import statements present in the file to show the model the available modules. We also extend dependencies to include the in-file dependent signatures.

Checking successful proofs. We defined a proof as successfully generated by the LLM if and only if SerAPI's `sertop` program accepted the proof when provided with (1) all lines in the file preceding the theorem (*i.e., the in-file context*), (2) the theorem's signature, and (3) the LLM-generated proof body. This validation was performed using the version of SerAPI that matches the Rocq Prover version used in the corresponding codebase.

5 Evaluation Results

We now share our evaluation results and use them to answer the four research questions we proposed in Section 1.

RQ1: How do external dependencies and/or context in the same source file impact proof generation for a theorem? Among the four ablations we introduced in Section 4, most models achieved the highest success rate in

[5]Our experiments were conducted on Verdi corresponding to commit `fdb4ede19d2150c254f0ebcfbed4fb9547a734b0`.

the informed mode, as shown in Tables 1a and 1b. For both `hs-to-coq` and Verdi, success rates dropped for most models when either in-file context or dependencies were excluded, with the worst results occurring when both were excluded.

One potential consequence of including all dependencies and in-file context is an increase in input tokens. To understand this implication, we also estimated the number of tokens required in both projects. We show the statistics in Table 2.

RQ2: How do LLMs perform on proofs of different sizes? Figures 4a and 4b show the proof generation success rates in each tactic count interval in light colors. These figures show that, with the exception of GPT-4o-mini, all LLMs have high success rates in generating proofs of small sizes. These success rates drop as the proof size increases. However, even when the proof becomes quite large, LLMs can still succeed in some cases in both projects.

However, one question we need to address to make sure our results are valid is to check whether LLMs were generating these proofs or whether they have simply "memorized" all these proofs, as both projects are open-source projects available online. For this reason, we further checked if the generated proofs are identical to the original proofs. We show all the generated identical proofs, or "plagiarized" proofs, in Figs. 4a and 4b using dark colors.

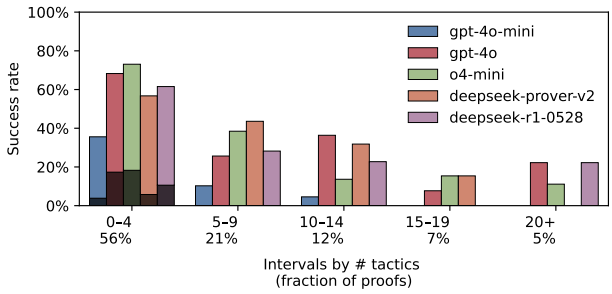
The results show that LLMs indeed generate identical proofs in both projects. In `hs-to-coq`, these are all small proofs, which have a high likelihood of being identical "by coincidence". On the other hand, some of the larger generated proofs in Verdi are identical to the original proofs, suggesting that the proof might have been in these models' knowledge set.

RQ3: Is there a difference when running LLMs on different verification projects? First, the impact of adding dependencies or in-file context also varies between these two projects. As seen in Table 3a, the benefits of in-file context diminished in `hs-to-coq` for proofs involving a larger number of tactics, and, in some cases, even reduced success rates for certain models. Conversely, simpler proofs with fewer tactics appeared to benefit from the in-file context.

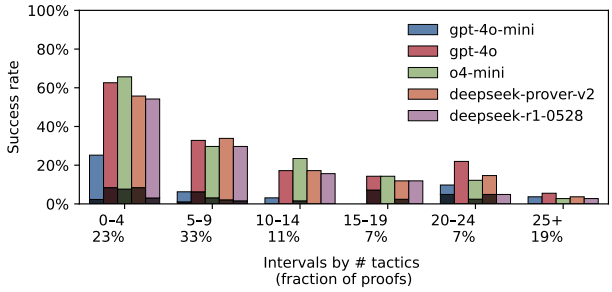
In contrast, for Verdi, adding in-file context had a remarkably strong effect. As shown in Table 3b, external dependencies alone were mostly insufficient for handling longer proofs (*e.g.,* 20+ tactics) with a higher number of tactics in the original proof. The models were only able to perform better in the informed mode, where the in-file context was provided.

Another difference between these two projects is that LLMs did not generate any proofs identical to original proofs in proofs with a larger tactic count in `hs-to-coq`.

It is unclear why `hs-to-coq` and Verdi exhibit these differences. However, this finding suggests that studying one project may not be sufficient for improving LLMs' effectiveness in other projects.



(a) `hs-to-coq`



(b) Verdi

Figure 4. Success rates (light) vs. identically generated proofs (dark) by tactic count intervals for `hs-to-coq` and Verdi.

RQ4: How is the quality of proofs generated by LLMs? In this section, we highlight some of the interesting proofs—including both successful ones and failed ones—generated by LLMs in our case study.

We compare the number of tactics in original proofs and in proofs generated by LLMs. In both projects, we find that LLMs can generate shorter proofs than the original ones.

Let's start with an example in `hs-to-coq`. We show an original proof demonstrating that `unit` is a monoid that satisfies all the `Monoid` typeclass laws in Fig. 5a. The theorem statement itself is not important. The original proof works by first `splitting` the theorem into four subgoals, each representing one monoid property. The proof then unfolds a number of definitions—a style that is consistent with many other proofs in the same file. Then, for each subgoal, the proof proceeds by either a case analysis or an induction.

We show a proof generated by OpenAI o4-mini and DeepSeek-R1-0528 with *no* external dependency or same-file context in Fig. 5b. The proof is much simpler: it first uses the `constructor` tactic, which does the same thing as `split` in the original here. Then, LLMs "realize" that all subgoals can be solved using the same sequence of tactics: `intros []` to introduce a variable into the context *and* perform a case

Table 1. Success counts and rates across different settings for `hs-to-coq` and Verdi.

(a) `hs-to-coq` (187 theorems)

Model	Informed	No in-file context	No dependencies	Neither
GPT-4o-mini	42 (22.5%)	35 (18.7%)	44 (23.5%)	31 (16.6%)
GPT-4o	92 (49.2%)	73 (39.0%)	83 (44.4%)	48 (25.7%)
o4-mini	97 (51.9%)	81 (43.3%)	81 (43.3%)	52 (27.8%)
DeepSeek Prover V2	85 (45.5%)	76 (40.6%)	71 (38.0%)	58 (31.0%)
DeepSeek R1	82 (43.9%)	74 (39.6%)	84 (44.9%)	48 (25.7%)

(b) Verdi (579 theorems)

Model	Informed	No in-file context	No dependencies	Neither
GPT-4o-mini	55 (9.5%)	27 (4.7%)	57 (9.8%)	17 (2.9%)
GPT-4o	177 (30.6%)	117 (20.2%)	170 (29.4%)	38 (6.6%)
o4-mini	172 (29.7%)	124 (21.4%)	177 (30.6%)	45 (7.8%)
DeepSeek Prover V2	164 (28.3%)	108 (18.7%)	159 (27.5%)	42 (7.3%)
DeepSeek R1	148 (25.6%)	123 (21.2%)	140 (24.2%)	40 (6.9%)

Table 2. Estimated prompt token counts for each setting, excluding the system prompt (rounded to the nearest integer). The token counts were estimated using OpenAI's TikToken library [65].

Project	Condition	Mean	Median	Max
`hs-to-coq`	Informed	3379	3162	10223
	No dependencies	1766	1292	6833
	No in-file context	1916	1862	5720
	Neither	152	147	228
`Verdi`	Informed	6944	5488	25357
	No dependencies	5653	4393	19289
	No in-file context	2559	1618	20674
	Neither	174	167	445

analysis on that variable at the same time, then `auto` for automatically discharging each goal.

An even smarter proof generated by LLMs can be found in Verdi. We show the theorem statement in Fig. 6. The theorem describes a relation between two variables, `failed` and `net`, when they are both in a multi-step transition relation defined by `step_ordered_dynamic_failure_star`—the exact definition of this step relation is not important. The original proof in Verdi is 24 lines of proof script, involves an induction, and Ltac's `match` statement.

However, OpenAI o4-mini is able to find a proof that consists of only 4 basic tactics on the informed mode, as shown in Fig. 6. This is because the contrapositive of this proposition has already been proven as a theorem right before this theorem (called `ordered_dynamic_failed_then_initialized`). OpenAI o4-mini "recognizes" this connection between the two theorems and proves this theorem by simply `applying` its contrapositive.

```
Instance instance_MonoidLaws_unit :
  MonoidLaws unit.
Proof.
  split;
    unfold op_zlzlzgzg__, Semigroup__unit,
         op_zlzlzgzg____,
         Base.Semigroup__unit_op_zlzlzgzg__;
    unfold mappend, mempty, mconcat,
         Monoid__unit, mappend__,  mconcat__,
         Base.Monoid__unit_mappend,
         Base.Monoid__unit_mempty,
         Base.Monoid__unit_mconcat.
  - intro x. destruct x. auto.
  - intro x. destruct x. auto.
  - intros. auto.
  - intros x. induction x; simpl. auto. auto.
Qed.
```

(a) The original proof showing that `unit` is a monoid that satisfies all the `Monoid` typeclass laws.

```
Proof.
  constructor; intros []; auto.
Qed.
```

(b) A proof for the same theorem generated by OpenAI o4-mini and DeepSeek-R1-0528. The two models generate the same proof for this theorem.

Figure 5. A comparison between the original proof and a proof generated by LLMs for theorem `instance_MonoidLaws_unit` in `hs-to-coq`.

Table 3. Percent gain in success rates from no in-file context (dependencies only) to informed per model and interval (with interval share in %).

(a) `hs-to-coq`

Model	0-4 (56%)	5-9 (21%)	10-14 (12%)	15-19 (7%)	20+ (5%)
GPT-4o	16.4	0.0	9.1	-7.7	11.1
GPT-4o-mini	8.7	-2.5	0.0	-7.7	0.0
o4-mini	14.4	10.3	-13.7	7.7	-11.1
DeepSeek Prover V2	0.0	18.0	9.1	0.0	0.0
DeepSeek R1	4.8	10.3	0.0	-15.4	11.1

(b) Verdi

Model	0-4 (23%)	5-9 (33%)	10-14 (11%)	15-19 (7%)	20-24 (7%)	25+ (19%)
GPT-4o-mini	7.6	4.1	3.1	0.0	9.8	3.7
GPT-4o	9.9	11.4	7.8	11.9	22.0	5.5
o4-mini	10.6	6.8	12.5	11.9	12.2	2.8
DeepSeek Prover V2	3.8	15.1	12.5	9.5	14.6	3.7
DeepSeek R1	4.6	4.7	3.1	7.1	4.9	2.8

```
Lemma ordered_dynamic_state_not_initialized_not_failed :
forall net failed tr,
  step_ordered_dynamic_failure_star
    step_ordered_dynamic_failure_init
    (failed, net) tr ->
  forall n, ~ In n (odnwNodes net) ->
  ~ In n failed.
(* The following proof is generated by OpenAI o4-mini. *)
Proof.
  intros net failed tr Hstar n Hnot Hin.
  apply Hnot.
  eapply ordered_dynamic_failed_then_initialized; eauto.
Qed.
```

Figure 6. A Rocq theorem found in Verdi (in the file `core/DynamicNetLemmas.v`) and a proof generated by OpenAI o4-mini. We omit the original proof found in Verdi because the proof script is 29 tactics long.

We should point out that the two theorems shown in Figs. 5 and 6 can also be solved using classical tools like CoqHammer [14, 15]. CoqHammer can solve the `hs-to-coq` theorem (Fig. 5) with its own tactic called `sfirstorder`. For the Verdi theorem (Fig. 6), it performs a proof search using an external automated theorem prover and also finds that the theorem can be proven with the help of its contrapositive, similar to the proof generated by LLMs. Nevertheless, it is impressive that LLMs are able to find these simple proofs given only one shot without a feedback loop.

The next theorem that LLMs come up with a simpler proof is the most surprising to us, and the theorem cannot be solved by CoqHammer. We show the theorem and its original proof in Fig. 7a. The theorem states that the pair `a * b` staisfy the `EqExact` law (Fig. 3) if both `a` and `b` satisfies this law. We show the original proof script in Fig. 7a to demonstrate the complexity of the original proof and to compare it with a proof generated by LLMs, but the reader should not try to read the proof script without Rocq Prover's interactive environment. The key structure of the proof is to perform two case analyses indicated by the two uses of the `destruct` tactic: (1) if two variables of type `a` are equal by `==`, and (2) if two variables of type `b` are equal by `==`.

We show a proof generated by DeepSeek-R1-0528 with the same-file context but *without* external dependencies in Fig. 7b. The proof is more concise. A main reason is that, instead of doing a case analysis on `==` like in the original proof, this proof pesforms a case analysis on `Eq_eq`, whose type is an inductively defined proposition `reflect` (Fig. 3) that relates `==` and `=`. Intuitively, performing a case analysis on `reflect` gives us more information than just `==`. For example, the original proof uses `move /Eq_eq` to convert between `==` and `=` in various places, but such a step is unnecessary in the generated proof.

Performing a case analyses on an inductively defined proposition like `reflect` to "extract more information" is a classical technique in mechanized verification discussed in Rocq Prover textbooks [66, Chapter "Inductively Defined Propositions"]. However, `Eq_eq` is a function defined in `hs-to-coq`—to be more precise, in the same file as `EqExact_pair`—and a case analysis on `Eq_eq` was used only once in the same file before this theorem, but DeepSeek-R1-0528 is still able to generate a proof like this in one shot without a feedback loop.

```
Instance EqExact_pair {a b}
  `{EqExact a} `{EqExact b} : EqExact (a * b).
Proof.
  split; rewrite /op_zeze__ /op_zsze__
    /Eq_pair___ /op_zeze____ /op_zsze____.
  case =>[??] [??] //=. destruct (_ == _) eqn:?.
  - rewrite andb_true_l. move /Eq_eq in Heqb0.
    destruct (_ == _) eqn:?.
    + constructor. move /Eq_eq in Heqb1.
      subst. reflexivity.
    + constructor. move /Eq_eq in Heqb1.
      intro. apply Heqb1.
      inversion H5; reflexivity.
  - rewrite andb_false_l. constructor.
    move /Eq_eq in Heqb0. intro.
    inversion H5. apply Heqb0. assumption.
Qed.
```

(a) The original proof showing that the pair `a * b` satisfies the `EqExact` law (Fig. 3) whenever both `a` and `b` do.

```
Proof.
  split; unfold op_zeze__, op_zsze__,
    Eq_pair___, op_zeze____, op_zsze____
    => - [x1 y1] [x2 y2] /=.
  - case: (Eq_eq x1 x2) => [-> | NEQx];
    case: (Eq_eq y1 y2) => [-> | NEQy];
    constructor; congruence.
Qed.
```

(b) A proof for the same theorem generated by DeepSeek-R1-0528.

Figure 7. A comparison between the original proof and a proof generated by LLMs for theorem `instance_MonoidLaws_unit` in `hs-to-coq`.

```
Lemma simpl_list_cons_eq a `{Eq_ a} (x y :a) xs ys :
  (cons x xs) == (cons y ys) = (x == y) && (xs == ys).
Proof.
    unfold op_zeze__, op_zeze____, Eq_list.
    simpl. auto.
 Qed.
```

(a) The theorem `simpl_list_cons_eq` theorem in `hs-to-coq` and its original proof.

```
(* The following proof is generated by GPT 4o-mini. *)
Proof.
  unfold Eq___list, op_zeze__, op_zeze____, eqlist.
  rewrite /eqlist.
  destruct xs, ys; simpl.
  - rewrite /op_zeze__.
    rewrite /op_zeze__.
    rewrite /op_zeze__.
    (* The proof goes on by repeating [rewrite]s. *)
```

(b) A failed proof generated by GPT 4o-mini that repeats the same tactic.

Figure 8. A comparison between the original proof and a failed proof generated by LLMs for theorem `simpl_list_cons_eq` in `hs-to-coq`.

On the negative side, we find that LLMs can stutter in generating some proofs. We show a theorem in `hs-to-coq` and its original proof in Fig. 8a. The proof is simple and straightforward. However, GPT 4o-mini generates a failed proof in the informed mode shown in Fig. 8b. The proof fails at the first `unfold`. However, an odder phenomenon is that, after the first `rewrite /op_zeze__`, the proof just keeps repeating the same tactic, seemingly indefinitely.

This phenomenon is not common, but it has appeared in both `hs-to-coq` and Verdi, in different theorems, various LLMs, when invoking various tactics. However, we cannot deduce or conjecture what the reasons are for this to happen.

Limitations. In terms of design, our experimental setup focused exclusively on single-pass proof generation, without incorporating a feedback loop. Although we recorded errors produced by `sertop` for each LLM and setting for future work, we did not analyze the model's ability to respond to errors. Moreover, when analyzing dependencies, we limited our scope to files within the same Rocq project. While the models were given the import statements in each setting, the external libraries were not indexed. Consequently, we assumed that LLMs would have implicit knowledge of widely used libraries such as ssreflect, or StructTact, InfSeqExt, and Cheerios (for Verdi), but did not verify this directly.

A further limitation lies in our dataset, which, while containing a substantial number of theorems, covers only two Rocq projects. Rocq projects may naturally vary in their structure and organization, which may heavily impact the results for settings with one of the dependencies or in-file contexts.

Finally, our experiments were conducted using Rocq version 8.10.2 and 8.11.0, which are both relatively old. While this choice was necessary to ensure compatibility with the codebases we studied, it may impact the relevance of results for newer versions of Rocq if the LLMs we used were trained on more recent versions of the language.

6 Related Work

Benchmarks for proofs. CoqGym is a pioneer in providing an extensive Rocq benchmark for machine learning models [92], containing 71K proofs from 123 real-life projects. It has been used by various studies on proof automation, such as First and Brun [23], First et al. [24], Lu et al. [51]. These works are also an inspiration for the case studies presented

in this paper. However, one issue with CoqGym is that it relies on older versions of Rocq Prover. For this reason, more recent tools like the `CoqPilot` benchmarking framework choose to build their own datasets [44].

Outside Rocq Prover, there are many benchmarks for other proof assistants or formal-method tools, such as DafnyBench [50], LeanDojo [93], miniCodeProps [49], FVAPPS [20], VerifyThisBench [19], Verina [94], *etc.*

Proof automation. Proof automation has always been a goal in research on proof assistants. Most of these works rely on automated theorem provers (ATPs) like SAT/SMT solvers. For example, SMTCoq [5] uses SAT/SMT solvers to prove theorems and then reconstructs Rocq proofs from them. CoqHammer [14, 15] defines a set of automation tactics for dependent type theory, uses external ATPs to find a proof, and then constructs a proof using its automation tactics by taking *hints* from proofs found by ATPs. In this way, CoqHammer is able to construct Rocq proofs that use intuitionistic logic with the help of ATPs that work on classical logic.

Other proof automation tools like Tactician [7] use machine learning (but not LLMs) instead. It provides suggestions for the next tactic based on "previously written tactics". CoqGym, the benchmark for Rocq proofs, also includes a tool called ASTactic, which is trained on CoqGym and uses deep learning to generate proofs automatically [92]. Some more recent works in this area include Proverbot9001 [72], Passport [73], QEDCartographer [74], *etc.*

LLMs and proof assistants. There have been a few recent works that investigate the capabilities of LLMs in generating proofs for proof assistants. We have already discussed Lu et al. [51]'s study on Verdi in Section 3.2. Qin et al. [67] studied FSCQ, a verified file system [12]. They conjectured that one reason LLMs fail to generate proofs is that LLMs struggle to find relevant lemmas when too many lemmas are given in a prompt [67, Section 4.3].

There have also been many works that leverage the power of LLMs to build proof-automation tools. For example, Baldur uses fine-tuned LLMs to generate whole proofs for Isabelle/HOL [25]. Their evaluation of Baldur on the PISA dataset [36] further shows that LLMs outperform small-model-driven search-based methods. *PALM* builds on its observation on Verdi (Section 3.2) and uses a generate-then-repair approach that combines LLMs and symbolic methods (*e.g.,* CoqHammer [14, 15]) to generate Rocq proofs [51]. Draft, Sketch, and Prove (DSP) uses LLMs to generate a sketch of a formal proof and then uses ATPs to fill in the missing details in the sketch [37]. Some other works in this area include Hu et al. [32], Kasibatla et al. [41], Lin et al. [48], Thompson et al. [82], Zhang et al. [98], *etc.*

Premise selection for proof generation. Premise selection refers to the process of selecting relevant *premises*, such as definitions and lemmas [35]. This is a common process used by many proof-generation works. For example, *PALM* uses Term Frequency-Inverse Document Frequency (TF-IDF) [39] and k nearest neighbors (KNN) [21] to select relevant premises. `CoqPilot` selects premises based on "metrics such as distance from the generation target or similarity with other theorem statements" [44].

Our work takes a much simpler approach by directly including dependencies and in-file context in the prompt. Prior works like Baldur did a similar thing, but they only included in-file context [25, Section 2.3].

LLMs and math. LLMs have been studied extensively in the context of mathematics. Earlier research focuses on benchmarking LLMs with simple math reasoning tasks [6, 13, 101]. Recently, Olympiad-level math theorem proving has been successfully tackled by LLMs [2, 46, 83]. There has also been rapid progress in auto-formalizing mathematics [47, 57, 88].

7 Conclusion

In this paper, we conduct a case study based on two real-world Rocq projects: the `hs-to-coq` project and Verdi. Our case study shows that LLMs can be effective in generating whole proofs for program correctness theorems. More specifically, we show that external dependencies and in-file context can significantly help with proof generation. We also find that LLMs perform well on small proofs. While its effectiveness degrades when the proof size increases, there is still a decent chance for it to generate whole proofs. However, our study also shows that the effectiveness characteristics of LLMs differ in different verification projects, which suggests that studying one project may not be sufficient for improving LLMs' effectiveness in other projects. Finally, we find that LLMs can generate concise and smart proof scripts, can apply classical techniques to new definitions, but can also produce meaningless stuttering proofs for unknown reasons.

We believe that using LLMs for verification with proof assistants is a promising direction that deserves more attention. Program verification is suitable for tools like LLMs that are unpredictable and can hallucinate [33, 99]. First, proofs are *not* computational. A generated *inefficient* proof has little to no impact compared with a generated inefficient program. Second, the proof-checking mechanisms in proof assistants (*e.g.,* `Qed` of Rocq Prover) can safeguard generated proofs to make sure that they are correct.

Verification with proof assistants can be potentially much more useful in software engineering if proof automation can be significantly improved. Indeed, researchers have argued that one major reason that formal methods are rarely used in software development today is their social aspect [27]. It will greatly improve the usability of formal methods (and hence the reliability of software) if LLMs can help with proof automation.

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